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CONTACT BEHAVIOR OF AN ORTHOTROPIC LAMINATED BEAM INDENTED BY A RIGID CYLINDER

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Abstract

A finite-element study of the contact behavior of laminated beams has been carried out. The influence of various parameters such as ply orientation, beam thickness, indenter size, presence of a compliant layer and the presence of matrix cracks and delamination on the contact behavior of a symmetric orthotropic laminate is studied. The results are expressed in terms of contact pressure distributions and force/indentation graphs. While laminate thickness, indenter size and the presence of a compliant layer affect the force/indentation behavior and pressure distributions, delamination seems to affect only the latter. Delaminations in the lower half, away from the contact regions, have an insignificant effect on contact behavior. Stacking sequence, except for the all 90° laminate, does not influence the contact behavior. © 1998 Elsevier Science Ltd. All rights reserved

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1 INTRODUCTION

Analysis and prediction of the response of fiber-reinforced composite laminates when subjected to transverse loading by a cylinder finds applications in low-velocity impact by foreign objects and in flexural tests for characterizing composite materials.

It is well known that by reinforcing polymers the strength-to-weight and stiffness-to-weight ratios are greatly enhanced in the direction of the fibers. However, the dynamic response of composite materials, when subjected to impact loading, is not satisfactory in general. This is because of the fact that the impact events involve relatively high contact forces acting on a small area. The history of the impact force produced during transverse impact is governed by various impactor parameters, such as impactor mass and velocity, as well as a suitable interaction law between the impactor and the composite structure. The form of the interaction (also known as contact law) is well known for isotropic materials, namely Hertz's contact law. However, in the case of aligned fiber-reinforced composites which are orthotropic in nature, the contact law is much more complex, particularly if the structure is in the form of a laminate. This is because the contacting lamina and the sequence of the other laminae in the laminate are additional parameters. These issues need to be adequately addressed.

Usually, in composites, a modified form of the Hertz contact law is used. Since Hertz's contact law is for semi-infinite solids, this modified law includes the effect of the properties of the target on the contact behavior through only the out-of-plane stiffness (modulus E_3) of that ply of the target which comes into contact with the impactor. It is felt, however, that the contact law should also depend on the thickness of plies, the relative orientation of various plies and various stiffness constants. This dependence on elastic constants other than E_3 has been shown by Keer and Schonberg[1] for a transversely isotropic beam.

Impact testing contact laws also find other applications. For example, in three- and four-point bending tests for testing the flexural and shear strengths of laminates, the stress distributions deviate significantly from those calculated from a strength-of-materials approach. A better understanding of the contact phenomena will explain these deviations since a flexural test is essentially equivalent to investigating a contact between a composite beam and a rigid indenter. Another area where these contact laws may be needed is tribology.

Earlier work in this area[1–3] was mainly concentrated on the contact behavior of orthotropic beams. An experimental study on $[90/45/90/-45/90]_{2s}$ and $[0/45/0/-45/0]_{2s}$ graphite/epoxy laminated beams indented by a spherical indenter was conducted by Tan and Sun.[4] They observed that the contact law

$$f = k\alpha^n$$

with $n = 1{\cdot}5$ during loading and $n = 2{\cdot}5$ during unloading, was satisfactory for most applications. The constant k is also different for loading and unloading. This contact law was used to solve impact problems with an average value of k. Cairns and Lagace[5] presented an analytical solution for thick composite plates subjected to lateral loading. The solution used a stress function approach to obtain the localized stresses and strains of a cylindrically orthotropic plate loaded by an axisymmetric rigid sphere. The properties of the laminate were the average of the properties of the laminae. The analysis agreed with the experimental data of Sun and Tan. Wu *et al.*[2] developed an analytical method to study the contact behavior of an orthotropic laminate indented by a frictionless cylinder. The method was derived from an exact Green's function and was based on anisotropic elasticity theory in the plane-strain condition. Unlike earlier papers they did not average the lamina properties. Two configurations, [0/90/0] and [0], were considered and it was shown that ply configuration did not affect the force/indentation behavior.

Although the aforementioned studies have yielded valuable insight into the contact mechanics of composite media, information about the effects of layer thickness, ply arrangement and indentor radius is rather sparse. In impact studies, matrix cracking and delamination are common damage modes. Introduction of a softer layer between laminae in a fiber-reinforced composite is one method of increasing the interlaminar fracture strength of the laminate. The effect of matrix cracking, delamination and the introduction of a softer layer on contact behavior has not received much attention, and needs to be addressed.

The objective of the present investigation therefore is to provide a comprehensive finite-element solution for a laminated beam loaded by a rigid disc. Force/indentation, force/contact-length relationships and pressure distributions in orthotropic laminates have been studied. The effects of ply arrangement, thickness of individual plies, indenter radius and the introduction of an isotropic layer in the laminate have been investigated. The effects of matrix cracking and delamination on indentation and contact pressure distribution are also analysed.

2 PROBLEM FORMULATION

The lamina is the basic building block of the laminated composite structure. The lamina is treated as a homogeneous, anisotropic material with specified average mechanical properties. The macroscopic relationship between stress and strain for a linearly elastic, anisotropic material is known as the generalized Hooke's law, and is given by

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl}$$

where σ_{ij} is a second-order stress tensor and C_{ijkl} is a fourth-order stiffness tensor. The stress and strain tensors are symmetric and contain only six independent quantities. The stiffness tensor for a general anisotropic, elastic material is also symmetric and contains 21 independent elastic constants. The number of independent constants in the stiffness tensor will decrease with the introduction of material symmetry. In view of the symmetry of σ_{ij}, ϵ_{kl}, and C_{ijkl}, the relationship between stresses and strains can be written in contracted notation as

$$\sigma_i = C_{ij}\epsilon_j \quad i,j = 1,\ldots,6$$

where C_{ij} is now referred to as a stiffness matrix.

The constitutive law for a transversely isotropic material in the coordinate system formed by the intersections of the three mutually orthogonal planes of material symmetry is given by

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{12} & C_{23} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix}$$

where there is no coupling between the normal stress components and the shearing strains. For the case of two-dimensional (2D) plane stress this constitutive law reduces to

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{12} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix}$$

Here, 1 and 2 are principal material directions parallel to the x and y axis, respectively.

These constants are related to the engineering constants by the following relationships

$$C_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, \quad C_{12} = \frac{\nu_{21}E_2}{1-\nu_{12}\nu_{21}},$$
$$C_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}, \quad C_{66} = G_{12}$$

In the present investigation, a 2D plane-deformation formulation is used. The equilibrium equations governing the distribution of stress in a plane are

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$$

$$\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0$$

The relationship between the strains and displacements in a body are given by

$$\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

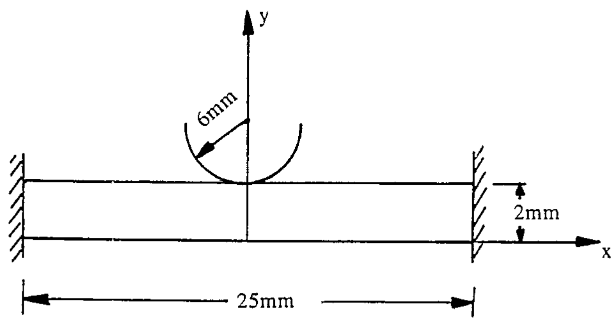

Fig. 1. Clamped beam indented by a rigid cylinder.

Our interest is in determining the surface displacements $u_2(0,0)$ in the y direction.

2.1 Boundary conditions and contact constraints

1. $x=0$ is the axis of symmetry.
2. The laminated beam is fixed at the ends. Any other boundary condition can be used. It was observed that since the contact length was one-tenth of the beam span the boundary conditions did not affect the contact behavior.
3. In order to prevent the contact surfaces from overlapping, an impenetrability condition which is satisfied at all times along the contact interfaces is used. This condition requires that the shortest distance (defined as gap, g) between two contact surfaces must be greater than or equal to zero. The contact constraint for normal contact are

$$\lambda_N \leq 0$$

$$g \geq 0$$

$$\lambda_N g = 0$$

where λ_N is the normal traction force at the contact point. These are the Kuhn–Tucker conditions for normal contact. The three equations reflect, respectively, the compressive normal traction constraint, the impenetrability constraint, and the requirement that the pressure is non-zero only when $g=0$.

2.2 Modelling of small sliding contact

2.2.1 Model and loading

An epoxy/graphite cross-ply laminate 25 mm long, 2 mm thick and clamped at the ends, as shown in Fig. 1, is considered. A rigid cylindrical disc acts at the center on the upper surface of the laminate. Plane-stress condition is used. The laminate is made up of eight unidirectional graphite/epoxy laminae. Because of the symmetry of the problem, we need to model only half of the laminate and cylinder. A fine mesh is used near the expected contact region in order to predict the contact pressure and contact area accurately. This is illustrated in Fig. 2. This refinement is accomplished by using the multi-point constraint provided in the ABAQUS[6] finite-element package for this purpose.

The eight-layered composite laminate is modeled with the PATRAN finite-element package. A graded mesh with 100 elements along the length in the expected contact region is used. This expected contact length is 1·25 mm. Each lamina is divided into five layers in the y direction and a graded mesh is used in this direction also. There are a total of 1725 eight-noded elements in the deformable slave surface.

A downward load of 800 N is applied at the center of the indenter. This load is applied in increments of 20 N. Here, we restrict ourselves to the small-contact stage so that we do not get into the region where indentation starts decreasing as the contact force is increased, as reported by Sankar[7] and Wu *et al.*[1]

ABAQUS has a good capability for studying surface contact behavior and offers two different methods for modeling the contact: small sliding and finite sliding

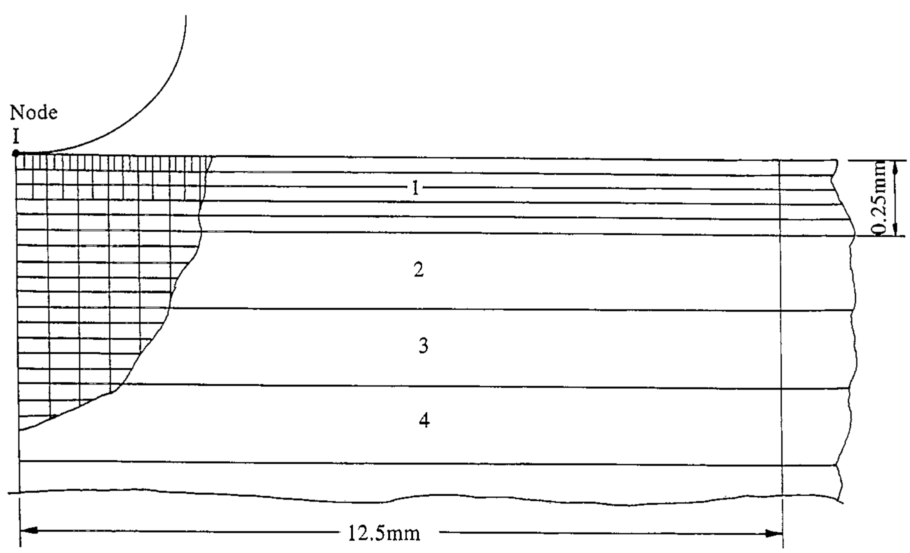

Fig. 2. Discretization in the contact region.

formulations. The small sliding approach implemented here uses the Lagrange formulation and the contacting surfaces can only undergo relatively small sliding relative to each other, but arbitrary rotation of the bodies is permitted. Here, of the two contacting surfaces, the composite laminate (the slave) is deformable and the cylinder (the master) is rigid. The nodes of the slave surface are constrained not to penetrate into the master surface. Consequently, the contact direction is always normal to the master surface. The contact between the surface is hard contact, i.e. when the surfaces are in contact any pressure stress can be transmitted between the surfaces with no penetration of one surface into another. The surfaces separate if the pressure reduces to zero. By small sliding it is meant that a given slave node will interact with the same local area of the master surface during the history of the analysis.

In the subsequent analysis, T300/934 graphite/epoxy is used. The material properties of the unidirectional lamina are $E_{11}=140$ GPa, $E_{22}=10$ GPa, $G_{23}=5{\cdot}68$ GPa, $G_{12}=3{\cdot}84$ GPa and $\nu_{12}=0{\cdot}3$.

3 RESULTS AND DISCUSSION

The effects of various parameters on the force/indentation behavior are presented in this section. It is also intended to fit a curve of the form $F=k\alpha^n$, where k and n are constants dependent on the stacking sequence, indenter size, etc. F is the force and α is the indentation. Indentation is calculated as the difference between the center displacements at the top and bottom of the beam.

3.1 Effect of indenter size

Three different indenters of radii 3 mm, 6 mm and 9 mm were used. These sizes are used commonly in both three- and four-point flexural tests and in low-velocity impact testing of composite structures. The force/indentation behavior and the contact distance versus contact pressure distribution for the above indenter sizes and for $[0/90/0/90]_s$ stacking sequence are given in Figs 3 and 4 respectively. Figure 3 shows that as the indenter size increases the indentation decreases. The values of k in N mm^{-n} for these radii are 46 020·9, 31 449·8 and 18 455. The corresponding values for n are 1·364, 1·311 and 1·225. Indentations are also plotted for 12 mm and 15 mm indenters in Fig. 3. The contact region exceeded 1·25 mm at 400 N for these two indenter sizes and therefore k and n values were not fitted to these. A plot of contact pressure versus contact length in Fig. 4 shows that, while the contact length increases, the maximum contact pressure decreases and the contact pressure curve flattens out as radius of the indenter increases. This flattening out is to be expected because the total contact force is same for all cases and a decrease in maximum contact pressure must be compensated by an increase in contact area. Since it is easier for the beam to

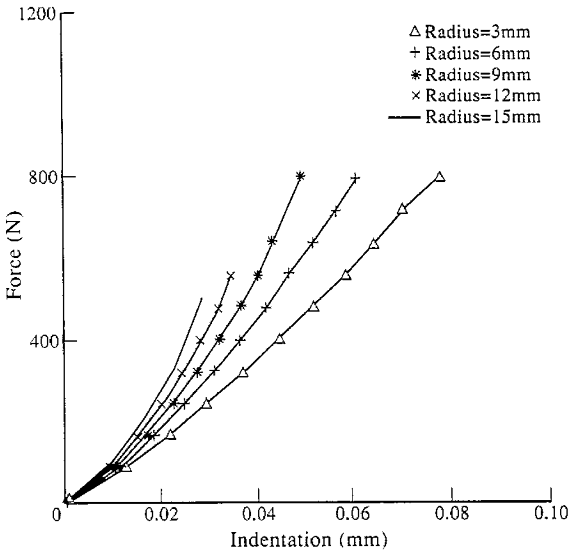

Fig. 3. Force/indentation relationship for different indentor radii for the $[0/90/0/90]_s$ laminate.

conform to a larger radius (and therefore smaller curvature) indenter, the peak pressures are lower and contact length larger as compared with the smaller indenter.

This behavior has been reported earlier for orthotropic beams[5] where it was shown that, for smaller indenters, the radius of curvature of the beam is much larger and the contact behavior is essentially that of the half plane. For a larger indenter the pressure in the central portion decreases as the beam wraps around the indenter.

Plots of σ_{yy} and τ_{xy} contours for $r=6$ mm indenter and 400 N load are given in Figs 5 and 6. In the contact region, σ_{yy} varies from a maximum of 531 MPa at the center to about 264 MPa near the contact edge. The maximum value of τ_{xy} is 149 MPa and is obtained 0·15 mm below the point where the contact ends.

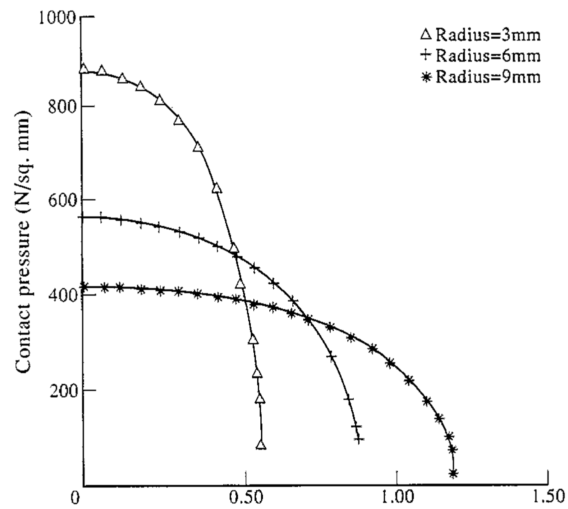

Fig. 4. Contact pressure distribution for different indentor radii for the $[0/90/0/90]_s$ laminate.

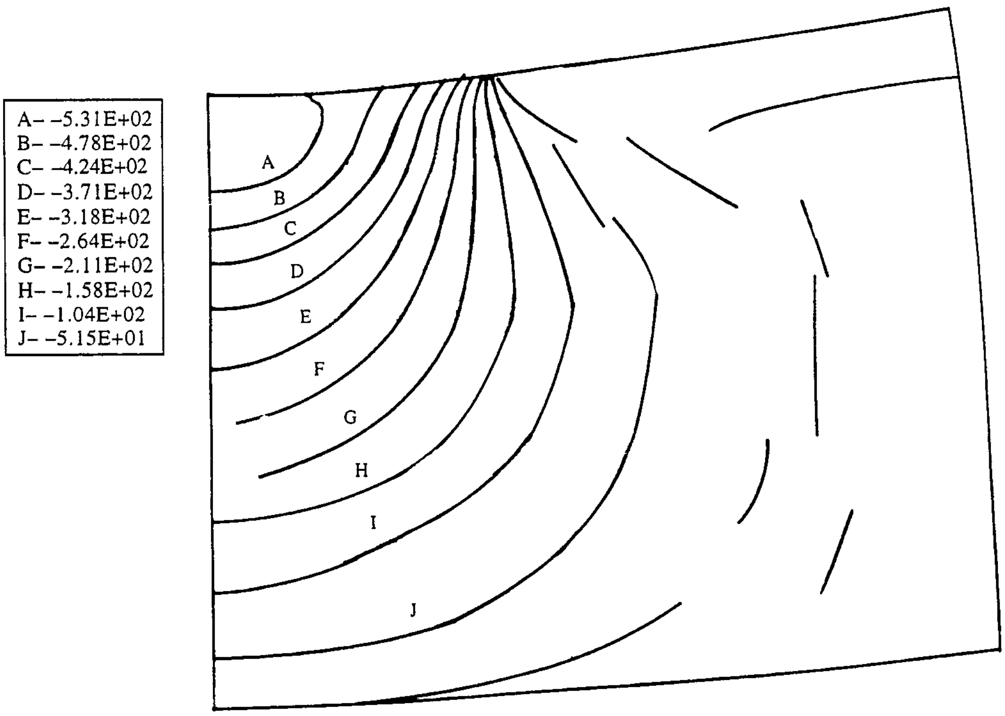

Fig. 5. Contour plots of stress σ_{yy} in the region below the contact for the $[0/90/0/90]_s$ laminate for a load of 400 N.

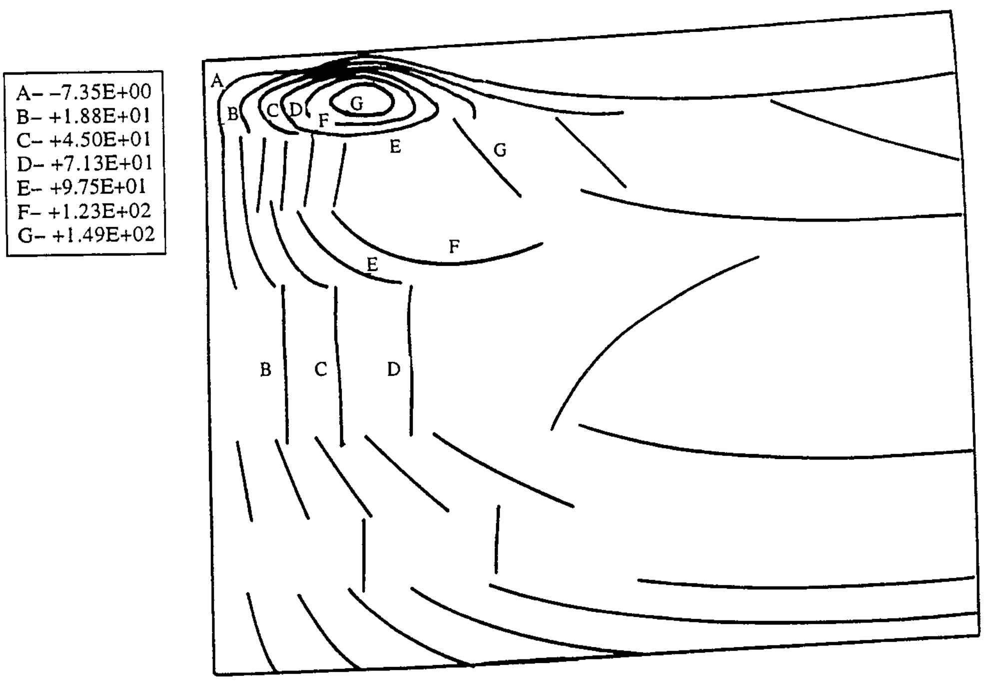

Fig. 6. Contour plots of stress σ_{xy} in the region below the contact.

3.2 Effect of stacking sequence

The effect of stacking sequence on the force/indentation behavior and the contact distance versus contact pressure relationship for an indenter size of $r = 6$ mm are given in Figs 7 and 8, respectively. Except for the laminate with all 90° laminae, the other laminates considered show no significant variation in the force/indentation behavior. For the laminate with all 90° laminae the contact length at 800 N is more than the predefined contact length of 1·25 mm and therefore results are presented only up to a load of 480 N. On comparison, for loads up to 480 N, the contact length

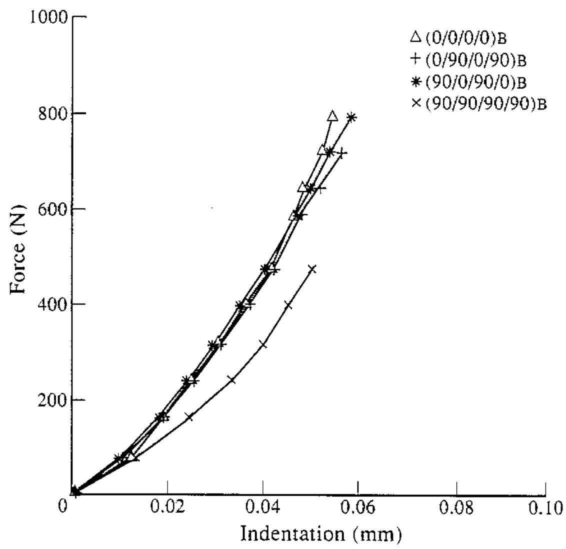

Fig. 7. Force/indentation relationship for different stacking sequences at a load of 480 N.

and indentation for all 90° laminae is always larger than for the other cases.

The laminate with all 0° laminae shows the highest maximum contact pressure and the lowest contact length. For the $[0/90/0/90]_s$ laminate the maximum contact pressure is slightly lower and the contact length is slightly longer. For the $[90/0/90/0]_s$ laminate the contact pressure decreases further and the contact length increases. The contact pressure is the lowest and the contact length the largest for the all 90° laminate. In general, the lower the flexural modulus, the easier it is for the beam to conform to the indenter shape and therefore the lower are the peak pressures.

Indentation cannot be explained on the basis of this conformity. For example, unlike a larger indenter size causing lower indentation and lower peak pressures, a lower flexural modulus of the beam leads to larger indentation and lower peak pressures. While ply orientation does not seem to have a significant effect on indentation, the peak pressures for the $[90/0/90/0]_s$ laminate are about 20% lower compared with those of the all 0° laminate. The best-fit values of k (in $\mathrm{N\,mm^{-n}}$) and n, in the force/indentation law for the all 90° stacking sequence, are 24 636·3 and 1·338, respectively. For other cases k and n are respectively 32 666·4 and 1·311.

The pressure distributions only approximately follow the parabolic distribution

$$p = p_0\left[1 - \left(\frac{r}{r_c}\right)^2\right]^{0.5}$$

where r is the radius at which pressure is measured, r_c is half the contact length and p_0 is the pressure at the center.

3.3 Effect of laminate thickness

To study the effect of laminate thickness, three laminate thicknesses ($t = 1{\cdot}6$, 2·0 and 2·4 mm) are used. Since the range of thickness studied here is much smaller than that studied by Wu *et al.*, we never observe the large-contact stage where the force/indentation curve turns back. However, the results for the thicknesses studied here follow the same trend as reported by Wu *et al.* The greater the thickness of the laminate, the less is the curvature conformation between the indenter and the laminate, leading to larger indentation and higher local stress concentration as shown in Figs 9 and 10. As the laminate thickness increases there is an increase in contact pressure accompanied by a decrease in contact length. The values of k (in $\mathrm{N\,mm^{-n}}$) and n for the three

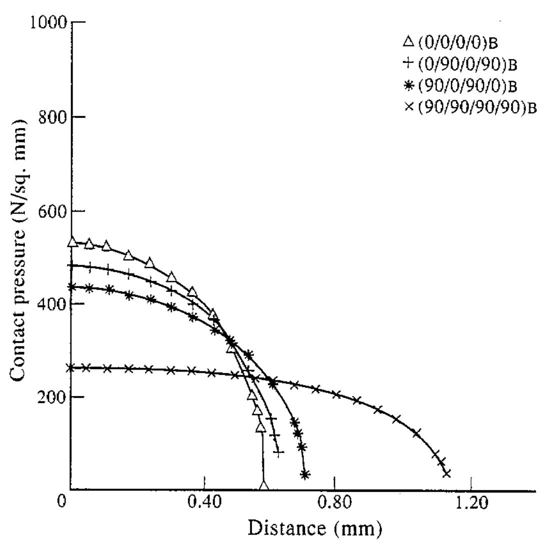

Fig. 8. Contact pressure distribution for different stacking sequences at a load of 480 N.

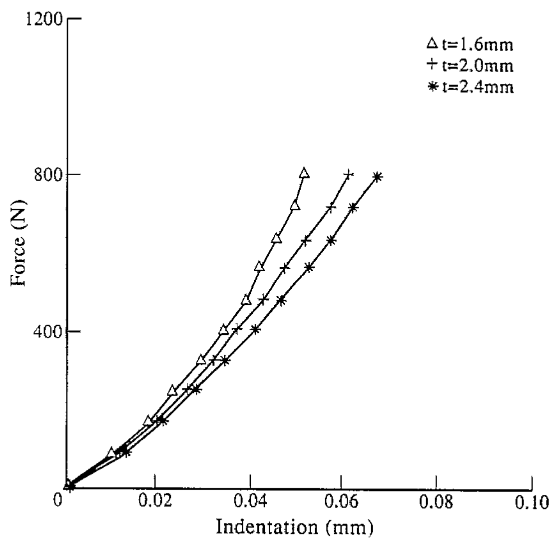

Fig. 9. Force/indentation relationship for different laminate thicknesses for the $[0/90/0/90]_s$ laminate.

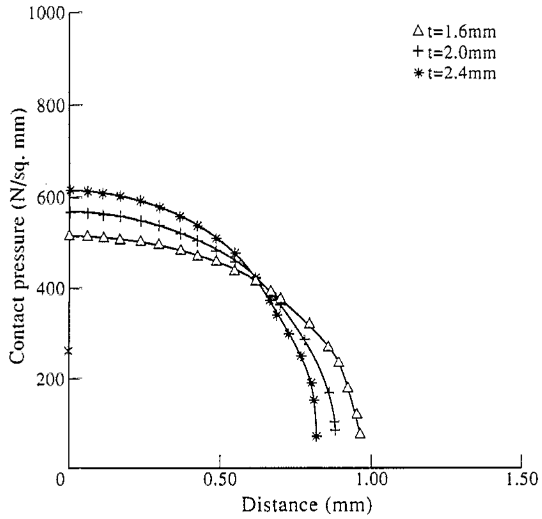

Fig. 10. Contact pressure distribution for different laminate thicknesses for the $[0/90/0/90]_s$ laminate.

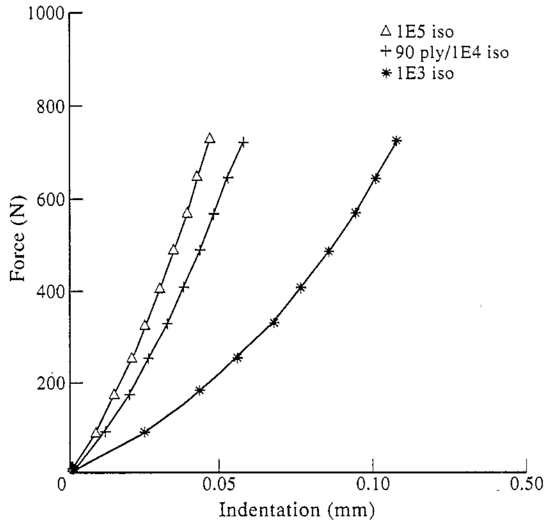

Fig. 11. Force/indentation relationship. The second layer and seventh layer of the $[0/90/0/90]_s$ laminate have been replaced by an isotropic layer. The curve of the isotropic layer with $E = 10^4\,\mathrm{N\,mm^{-2}}$ coincides with that of the original laminate and is not plotted.

thicknesses mentioned above are 45 293·5, 31 449·8, 25 772 and 1·373, 1·311, 1·283, respectively.

3.4 Effect of compliant layer

Multilayered composite laminates are typically constructed by bonding together a number of laminae with various material properties and fiber orientations. In the process of assembling the individual laminae, adhesives such as epoxy, which are isotropic and have low elastic modulus, are used to bond the layers firmly together. If the properties of this adhesive layer are very different from those of the individual plies it can substantially affect the overall properties of the laminate.[3] Also, interleaving is sometimes used as a method for increasing the interlaminar fracture strength of laminates. In this technique, thin films or 'interleaves' made of a tough polymer are embedded between the fiber-reinforced resin laminae.

The effect of varying material properties on the indentation of a laminated beam can be best illustrated by replacing two of the unidirectional laminae, one each above and below the midplane of the laminate, with isotropic layers bonded to the adjacent laminae. Here, we replace layer 2 of the 2 mm thick $[0/90/0/90]_s$ laminate by an isotropic layer whose elastic modulus, E, can be varied. Three values of E (10^3, 10^4 and $10^5\,\mathrm{N\,mm^{-2}}$) are used in the analysis. The force/indentation behavior and the contact distance versus contact pressure relationship for an indenter size of $r = 6$ mm are given in Figs 11 and 12. It is observed that introduction of a softer layer increases indentation and that there is a decrease in the peak contact pressure. This decrease in peak contact pressure is almost 30% and can probably explain the reduction of damage for composites with a compliant layer. A comparison of the stress distribution patterns in the presence of compliant layer did not show much variation from those shown in Fig. 5.

3.5 Delamination and matrix cracking

Delamination and matrix cracking are common damage modes in composites which occur either during manufacturing or during use. In impact problems they generally occur together with delaminations originating from the matrix cracks. The matrix cracks in these problems can either be shear cracks which are embedded in the laminate or bending cracks which appear in the outermost layers of the laminate. To study the effects of delamination and matrix cracking on contact behavior the $[0/90/0/90]_s$ stacking sequence is chosen. In one case, the crack was introduced in the second layer from the top and a delamination was introduced at the interface

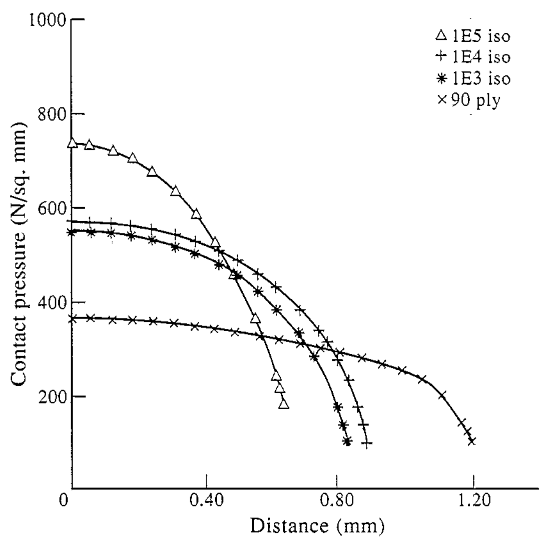

Fig. 12. Contact pressure distribution for the case of a compliant layer.

between this and the top 0° layer. In the second case the

minations. The presence of a crack and delamination in using different values of k and n. From this study it seems that, in the unloading region, the change in the coefficients k and n cannot be a consequence of a small delamination or crack.

4 CONCLUSION

A parametric study to find the effect of indenter size, stacking sequence, delamination, matrix cracking, laminate thickness and replacing a layer by an isotropic layer, on indentation, contact pressure, contact length for has been carried for a layered beam. The conclusions are tabulated in Table 1. From our study it appears that the contact law must include the effects of laminate thickness, but these are generally absent in contact laws based on a half plane. The presence of an isotropic layer with E lower than E_3 of the laminate reduces contact pressure considerably. The variation of k and n with thickness and indentor size are given for some cases and a definite pattern seems to emerge. Effects of friction have not been included in the present study. This may, however, be important in determining contact laws for oblique impact.

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REFERENCES

1. Keer, L. M. and Schonberg, W. P., Smooth indentation of a transversely isotropic cantilever beam. *Int. J. Solids Struct.*, 1986, **22**, 1033–1053.
2. Wu, E., Chao, J. and Yen, C., Smooth contact of orthotropic laminates by rigid cylinders. *AIAA J.*, 1993, **31**, 1916–1921.
3. Urquhart, E. E. and Pindera, M., Frictionless contact of rigid indenters on multilayered composite half planes. Interim Report, No. AM-93-02, University of Virginia, Charlottesville, Virginia, 1993.
4. Tan, T. M. and Sun, C. T., Use of statical indentation laws in the impact analysis of laminated composite plates. *J. Appl. Mech.*, 1985, **52**, 6–12.
5. Cairns, D. S. and Lagace, P. A., Thick composite plates subjected to lateral loading. *J. Appl. Mech.*, 1987, **54**, 611–616.
6. *ABAQUS/Standard User's Manual*, Version 5.4. Hibbitt, Karlsson and Sorenson, Inc., USA, 1994.
7. Sankar, B. V., Smooth indentation of orthotropic beams. *Compos. Sci. Technol.*, 1989, **34**, 95–111.