

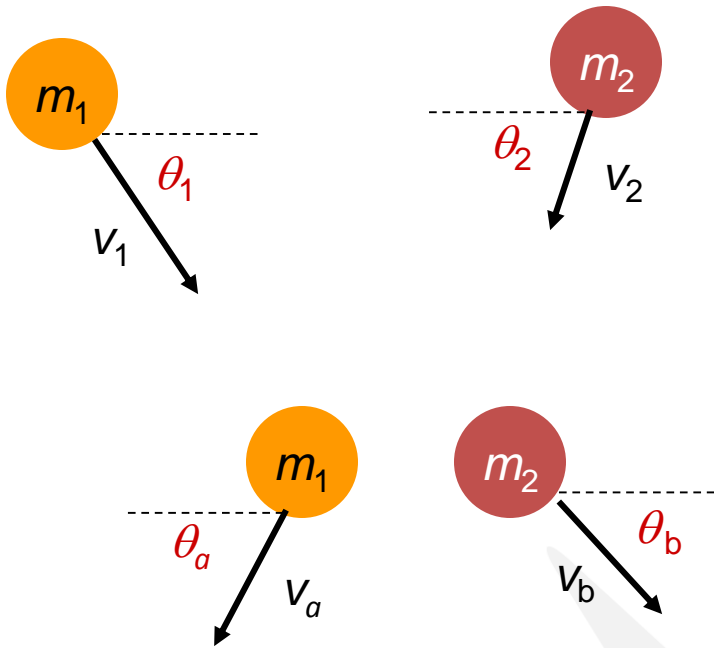
ESTIMATING ΔV

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Conservation of Momentum in 2-D



before:

$$p_x = m_1 v_1 \cos \theta_1 - m_2 v_2 \cos \theta_2$$

$$p_y = m_1 v_1 \sin \theta_1 + m_2 v_2 \sin \theta_2$$

after:

$$p_x = -m_1 v_a \cos \theta_a + m_2 v_b \cos \theta_b$$

$$p_y = m_1 v_a \sin \theta_a + m_2 v_b \sin \theta_b$$

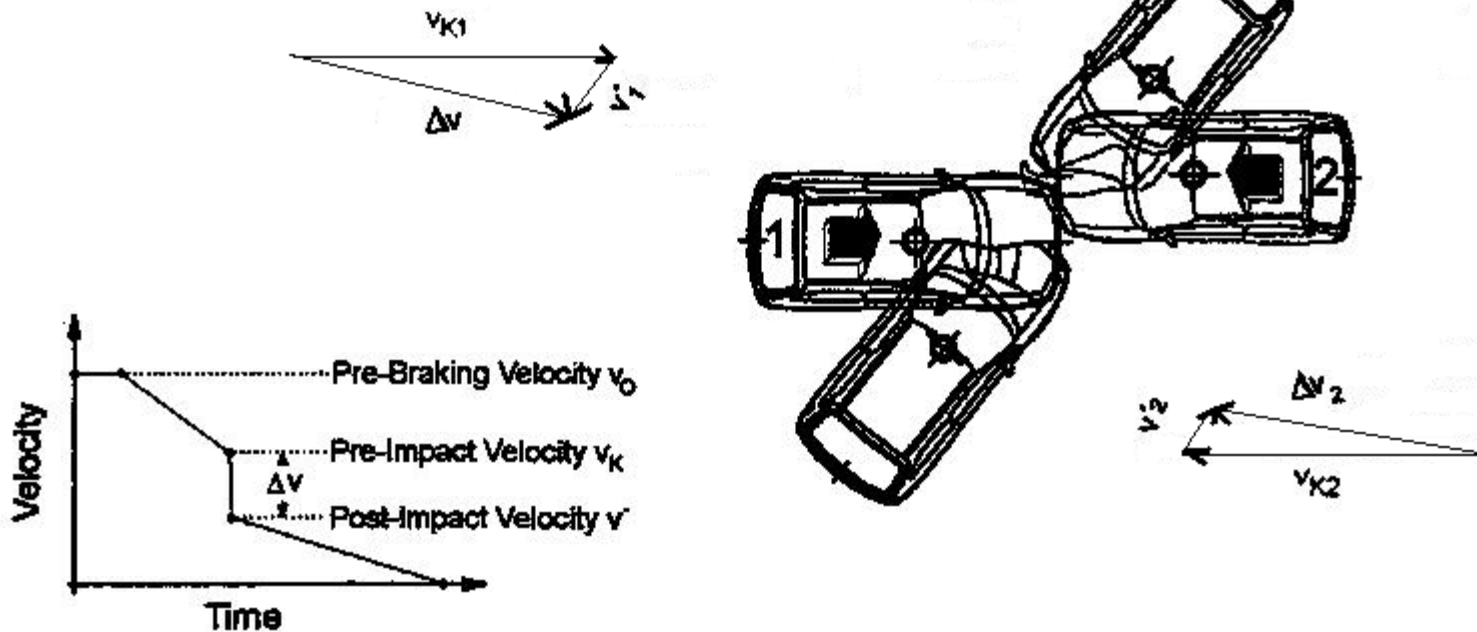
Conservation of momentum equations:

$$m_1 v_1 \cos \theta_1 - m_2 v_2 \cos \theta_2 = -m_1 v_a \cos \theta_a + m_2 v_b \cos \theta_b$$

$$m_1 v_1 \sin \theta_1 + m_2 v_2 \sin \theta_2 = m_1 v_a \sin \theta_a + m_2 v_b \sin \theta_b$$

What is DeltaV (ΔV)

- Delta-V is the change of velocity of the center of gravity of a vehicle in the road fixed coordinate system during the contact phase
- Different from the change of velocity calculated by the integration of the vehicle deceleration components measured in the Event Data Recorder (EDR)



CRASH3 and WinSMASH

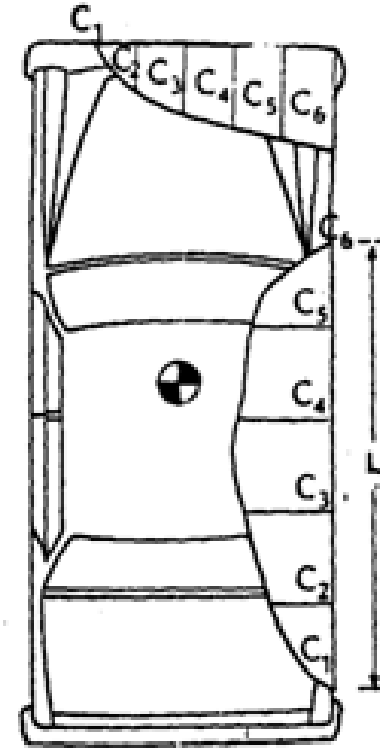
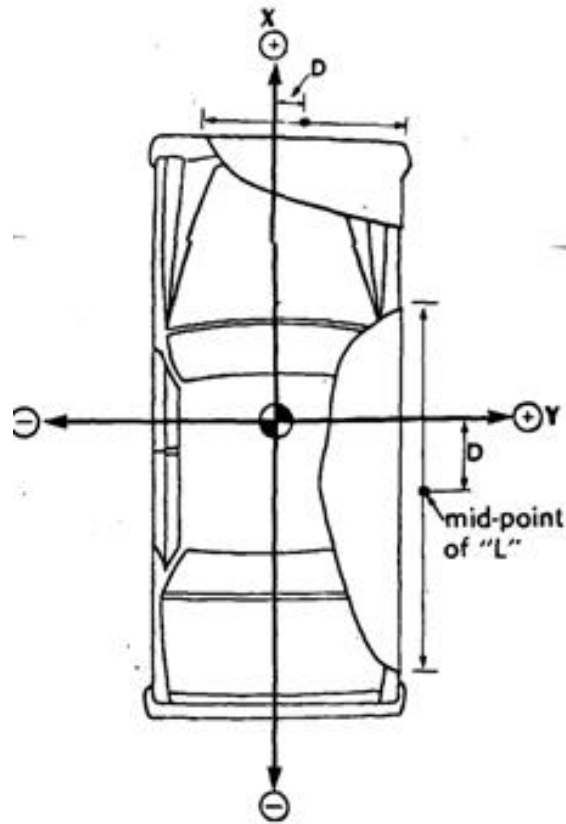
- The National Highway and Transport Safety Association (NHTSA) evolved CRASH3 in the 1980's and more recently WinSMASH
- Trajectory Analysis:
 - + Analyze the ΔV using detailed scene measurement and conservation of momentum
- Damage analysis
 - + ΔV has a linear relationship with residual damage
- Satisfactory agreement is supposed to exist between the two estimates when their delta-V components differ by no more than 4 kmph or ten percent, whichever is greater.
- In user friendly form available as AiDamage™
- Output is
 - + Three components-x, y and angular: ΔV
 - + Energy dissipated (mainly) in crushing the vehicular structure

Crush Measurement

- A measure of the energy absorbed and hence its effectiveness in fulfilling its design objective
- An estimate of the change in the speed of both vehicles (or one vehicle for say a tree or a barrier)
- Estimating the energy and delta-Vs
- Need a rig with lasers or use old fashioned tape and plumbline.



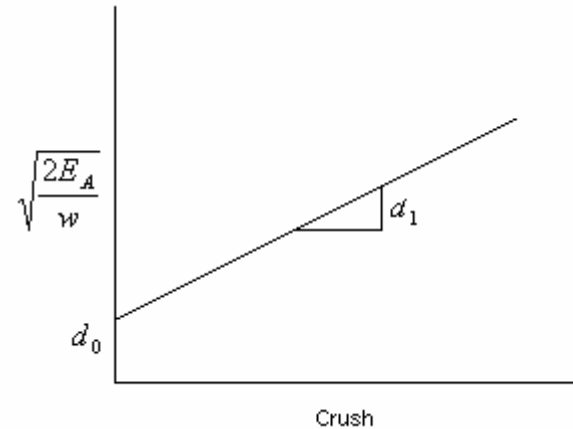
Make a sketch



- This data is often reported in the Collision Deformation Classification (CDC) as per SAE J224 MAR80

Unitized body cars

- Linear relationship between the impact velocity and crush was observed from the crash tests conducted on 1971-1974 General Motors full frame body cars.
- It has been experimentally established that residual crush is linearly proportional to $\sqrt{(2E_A/w)}$ for unitized body cars
- E_A is the energy absorbed by the vehicle structure and w is the width of the crush.



- Over a crush profile $c(w)$:
$$E_w = \int_0^w \frac{1}{2} (d_0 + d_1 \times C)^2 dw$$

Vehicle Data Needed

Type	Wheelbase
1	$\leq - 240.8$
2	240.8 – 258.0
3	258.0 – 280.4
4	280.4 – 298.4
5	298.4 – 312.9
6	$> - 312.9$
7(vans)	276.8 – 330.2

$k = 0.3 \times (\text{vehicle length})$

The generic d^0 and d^1 stiffness coefficients may not apply for bumper over-ride and under-ride crashes for example

Cat.	Front		Rear		Side	
	d_0	d_1	d_0	d_1	d_0	d_1
	$\sqrt{\text{Newton}}$	$\frac{\sqrt{\text{Newton}}}{\text{cm}}$	$\sqrt{\text{Newton}}$	$\frac{\sqrt{\text{Newton}}}{\text{cm}}$	$\sqrt{\text{Newton}}$	$\frac{\sqrt{\text{Newton}}}{\text{cm}}$
1	91.4	6.7	93.88	5.43	63.3	6.83
2	97.0	7.22	96.23	5.28	63.3	8.02
3	102.1	7.25	99.49	5.56	63.3	7.50
4	107.0	6.36	99.99	5.37	63.3	7.21
5	109.6	6.18	99.97	4.50	63.3	5.19
6	116.0	5.75	74.86	6.94	63.3	5.69
7 (vans)	109.7	8.51	98.69	7.79	-	-
8 (pickup)	105.7	7.98	101.42	7.77	-	-
9 (FWD)	99.18	6.46	-	-	-	-

Computing Delta V

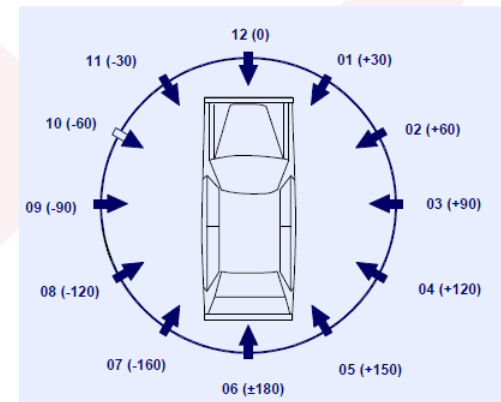
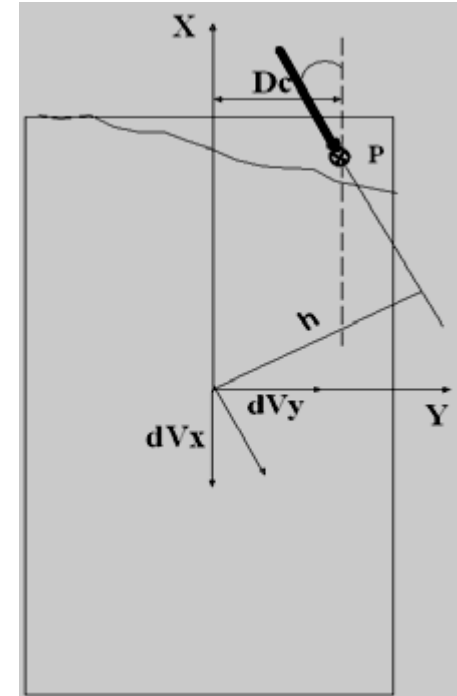
- The total energy, E_T = sum of energy absorbed by each vehicle.
- The radius of gyration of vehicles 1 and 2 are k_1 and k_2 and the moment arm of impact force h_1 and h_2 .

$$\Delta V_1 = \sqrt{\frac{2E_T \gamma_1}{M_1 \left(1 + \frac{\gamma_1 M_1}{\gamma_2 M_2}\right)}}$$

$$\gamma_1 = \frac{k_1^2}{k_1^2 + h_1^2}$$

$$\gamma_2 = \frac{k_2^2}{k_2^2 + h_2^2}$$

$$\Delta V_2 = \sqrt{\frac{2E_T \gamma_2}{M_2 \left(1 + \frac{\gamma_2 M_2}{\gamma_1 M_1}\right)}}$$



Help for you : modern packages

1. Impact velocity
2. Position of impact (POI) in x-y direction
3. Contact plane angle phi (degrees CCW from the global x-axis)
4. Pre-impact directions
5. Vehicle positions
6. Coefficient of Restitution
7. Contact plane friction

Minimise the weighted error of these parameters.

$$Q = \sqrt{\frac{\sum_i (W_i \cdot X_i)^2}{\sum_i W_i^2}} \cdot 100\%$$

Using Gauss-Sidel, GA or Monte Carlo in isolation or in combination

Summary

$$\Delta V_1 = \sqrt{\frac{2E_T \gamma_1}{M_1 \left(1 + \frac{\gamma_1 M_1}{\gamma_2 M_2}\right)}}$$

$$\Delta V_2 = \sqrt{\frac{2E_T \gamma_2}{M_2 \left(1 + \frac{\gamma_2 M_2}{\gamma_1 M_1}\right)}}$$

- Measure the crush profiles and centre of deformation.
- Estimate direction of approach to estimate h .
- Lookup values of m and k for both cars.
- Estimate d_0 and d_1 from the chart or measured data.
- Ready to find ΔV manually or using a package.
- Do remember this is a gross estimate and has to be verified and tuned through trajectory reconstruction.